Decision Making Process on Multi-Objective Optimization Results

Marian N. Velea and Simona Lache

***Abstract*—The interpretation of results in the case of a multi-objective optimization study is partially made with the help of the trade-off curves which indicate the set of solutions that gives the best compromise between objectives. However, there is a further need for choosing one single solution from this set. This article is discussing a decision making process for selecting the most beneficial design solution with respect to a set of proportion factors applied on the objectives and defined by the end-decision makers. The results show that better trade-offs may be obtained when solving the multi-objective problem using dedicated algorithms combined with the proposed selection method instead of reducing the problem to a single-objective definition by considering weight factors.**

***Index Terms*—Multi-objective optimization, Pareto set, negotiated design solutions.**

I. Introduction

In the design process, the identification of the optimal solution meeting the set of requirements is of utmost importance. Although single-objective optimization problems are nowadays relatively simply solved through dedicated methods, challenges still exist when more conflicting objectives are considered. For example, one of the engineering tasks where optimization is successfully used is weight reduction, since it represents one important measure to take in order to improve the engineering systems' performance. However, weight reduction usually implies the reduction of other performance criteria such as the stiffness and strength properties or the material cost. Therefore, there are cases where many objectives need to be defined and considered within the optimization procedure in which case a conflict situation appears between objectives; this is when an increased performance in one objective leads to a decreased performance for the others [1].

Several complex techniques and algorithms have been proposed for solving such multi-objective optimization problems [2]-[5]. The weighted sum approach has been used as an attempt to simplify the problem complexity of finding solutions within multi-objective optimization problems, where all the objectives functions are summed into a single objective function, giving weight penalties for each of them [4]. Then, a solution may be obtained by running one of the many existing single-objective optimization algorithms. The main drawback regarding the weighted sum method is represented by the quantification of weight penalties because the results are strongly dependent on them [6].

In order to obtain a large spectrum of solutions, dedicated multi-objective optimization algorithms are of interest [7]-[9]. One of the most spread algorithms within the current available commercial FE packages is the so-called MOGA (Multi-objective Optimization Genetic Algorithm). GRSM (Global Response Surface Method) algorithm is also used for solving multi-objective optimization problems recommended for large and time consuming models [10]. Instead of providing one single solution, the dedicated multi-objective optimization algorithms produce a set of solutions by searching within the design space for a set of Pareto optimal solutions [11]. The interpretation of results in the case of a multi-objective optimization study is partially made with the help of the trade-off curves, the so-called Pareto frontiers. The obtained Pareto frontiers only indicate the set of solutions that gives the best compromise between objectives, but there is a further need for choosing one single solution from the set. This can be done currently either by intuition or by reformulating the objectives as constraints, except one of them, or by using a composite objective function [1].

In order to deal with the difficulties of interpreting the multi-objective optimization results, this article is discussing a method in order to select the most beneficial design solutions with respect to a set of proportion factors applied on the objectives and defined by the end-decision makers, after running the optimization.

II. Method

In order to demonstrate the proposed method, let us suppose a simple multi-objective optimization problem where functions $f1$ and $f2$ are to be minimized, Eq. (1) and (2), and function $f3$ is to be maximized, Eq. (3). All these functions take values in terms of $x\in[1,100]$ which represents the design variable.

$$f_1 = \frac{x^2}{200}. \quad (1)$$

$$f_2 = \frac{100}{\sqrt{x}}. \quad (2)$$

$$f_3 = 20\, log\, x. \quad (3)$$

The above considered functions are graphically illustrated within Fig. 1.

The goal is to find the value of the design variable x that may offer the best trade-off solution between the conflicting objective functions in terms of proportion factors defined by

Manuscript received January 27, 2015; revised July 24, 2015. This work was partially supported by the strategic grant POSDRU/159/1.5/S/137070 (2014) of the Ministry of Labor, Family and Social Protection, Romania, co-financed by the European Social Fund – Investing in People, within the Sectorial Operational Program Human Resources Development 2007-2013.

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decision makers. The resulted Pareto frontiers are shown within Fig. 2 and Fig. 3. The trade-off between two objectives may be realized between the points that define the Pareto optimal set, in terms of the objective importance. However, when it comes to multiple objectives, difficulties with using the Pareto front arise from the fact that the best compromise between two objectives does not necessarily represent the best one between other two objectives. Therefore, an overall performance of the objectives is needed, that relates the contribution of each objective when searching the most beneficial overall solution.

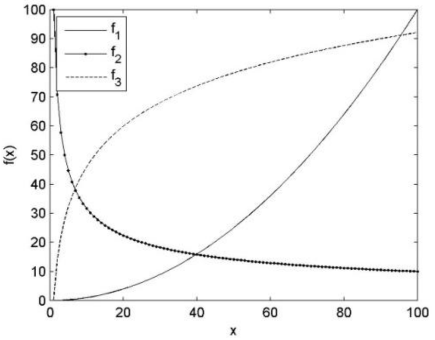


Fig. 1. Considered functions within the defined multi-objective optimisation problem.

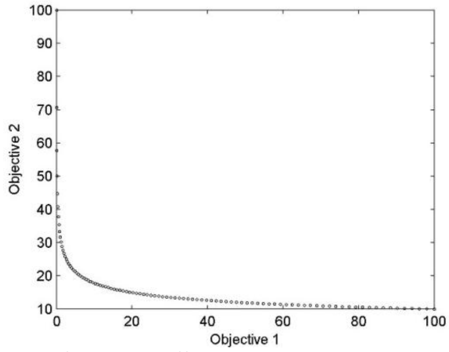


Fig. 2. Pareto front–trade-off solutions between Obj. 1 and Obj. 2.

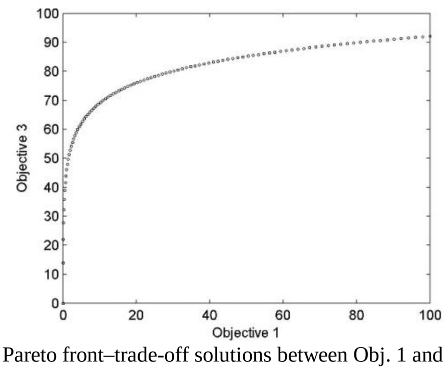


Fig. 3. Pareto front–trade-off solutions between Obj. 1 and Obj. 3.

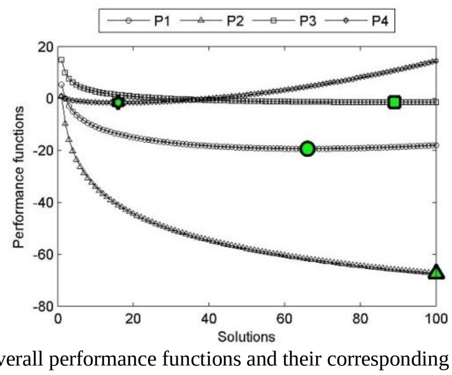


Fig. 4. Overall performance functions and their corresponding minimum values.

Such an overall performance function P is adapted here from [1], where proportion factors are applied to relate the contribution of the objectives, Eq. (4):

$$P = \frac{\sum_{i=1}^{n} p_i \times \xi_i - \sum_{j=1}^{m} p_j \times \xi_j}{100} \quad (4)$$

where:

p_i and p_j represents the proportions to which the value of the i^{th} objective (to be minimized) and the value of the j^{th} objective (to be maximized) contributes to the overall performance function P, in percent;

- $\sum_{i=1}^{n} p_i + \sum_{j=1}^{m} p_j = 100\%$.

- $\xi_i = \frac{(\overline{O_i} - O_{i_{min}})}{O_{i_{max}} - O_{i_{min}}}$, $\xi_i \in [0,1]$; $\xi_j = \frac{(\overline{O_j} - O_{j_{min}})}{O_{j_{max}} - O_{j_{min}}}$, $\xi_j \in [0,1]$

$\overline{O_i} = \frac{O_i}{O_{i_{ref}}}$, represents the normalized value of the i^{th} objective to the reference value, i=1, ..., n;

$\overline{O_j} = \frac{O_j}{O_{j_{ref}}}$, represents the normalized value of the j^{th} objective to the reference value, j=1, ..., m;

The objective's reference value is calculated by assuming an initial value for the design variable x_{init}: $O_{i_{ref}} = f_i(x_{init})$; $O_{j_{ref}} = f_j(x_{init})$

$$O_i = f_i(x) \,;\, O_j = f_j(x);$$

$O_{i_{min}}, O_{i_{max}}$ –minimum and maximum values of the objective to be minimized, i=1, ..., n;

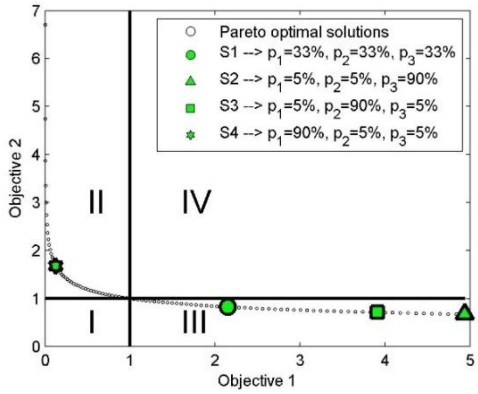
$O_{j_{min}}, O_{j_{max}}$ –minimum and maximum values of the objective to be minimized, j=1, ..., m;

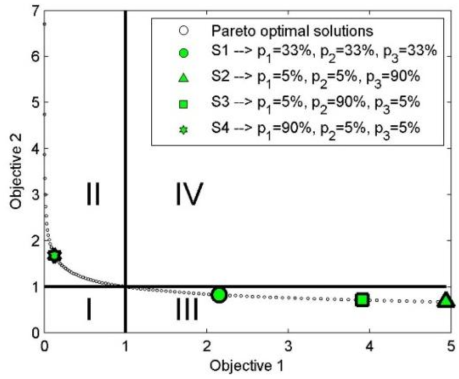
Fig. 4 shows the resulted overall performance functions P by considering 4 cases with different distributions of the proportion factors pi and *pj*, Table I, and an initial value of 45 for x. The minimum value of the overall performance functions P gives the best compromise between the considered objectives while taking into account the desired value for the proportions pi and *pj*.

The obtained values for the objective functions are graphically presented within Fig. 5 and Fig. 6. For the objectives to be minimized, improvements are observed if their normalized value is below 1, while a value above 1 indicates improvements of the objectives to be maximized. Table I, correlated with Fig. 5 and Fig. 6, shows a selection of four possible solutions obtained by varying the proportion factors p_i and p_j in such a way to give different contributions of the objective functions to the obtained design solution.

The plots shown, Fig. 5 and Fig. 6, are divided into four regions (I - IV) in order to clearly show and classify the performance offered by each of the design solutions. The performance is increased for both of the objectives if the solution comes from region I or it is decreased for both of the objectives if the solution comes from region IV. Region II and III contain those solutions where only one of the objectives has an increased performance.

The solution denoted $S1$ shows a tied trade-off between objectives, all having equal influence on the solution (33% each), Table I. *S*2 – *S*4 represent extreme dominated solutions: *S*2 represents *Objective 3* dominated solution (p = 90%). *S*3 represent *Objective 2* dominated solution (p = 90%). *S4* represent *Objective 1* dominated solution (p = 90%), Table I.





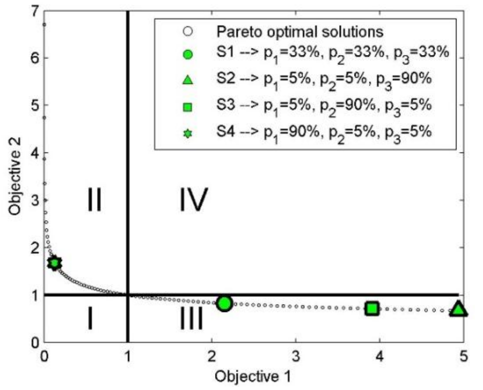


Fig. 5. Obj. 1 vs. Obj. 2 - Selected solutions using the herein proposed method.

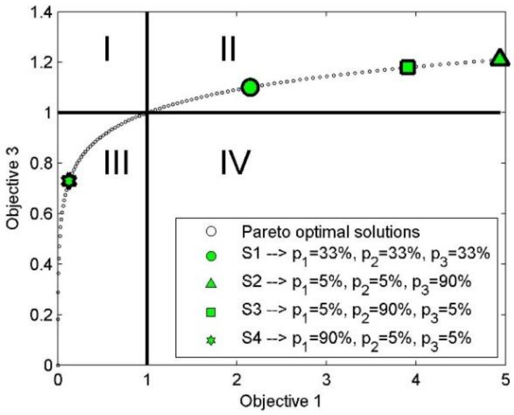


Fig. 6. Obj. 1 vs. Obj. 3 - Selected solutions using the herein proposed method.

TABLE I: RESPONSES' VALUES AND THEIR CORRESPONDING PROPORTION FACTORS FOR DIFFERENT DESIGN SOLUTIONS

Responses	**Solutions**							
	S1		**S2**		**S3**		**S4**	
	Obj. value	*p %*	*Obj. value*	*p %*	*Obj. value*	*p %*	*Obj. value*	*p %*
$f_1(x)$	2.15	33	4.94	5	3.91	5	**0.13**	**90**
$f_2(x)$	0.83	33	0.67	5	**0.71**	**90**	1.7	5
$f_3(x)$	1.1	33	**1.21**	**90**	1.18	5	0.73	5
P_s	-19.31		-67.43		-1.43		-1.62	
x	66		100		89		16	

III. CASE STUDY

The above presented method is applied further on within a simple case study. The problem consists in minimizing 3 conflicting objectives related to a cantilever beam, Fig. 7: min: *m* - mass of the beam; min δ_z – deflection at the free-end when applying the force F_z; min φ_x – rotation at the free-end when applying the torque *T*. The optimization model was defined within Excel and it was solved using MOGA within the Hyperstudy® facilities.

The optimization results may be interpreted from Fig. 8 and Fig. 9, correlated with Table II. The solution denoted *S1* represents a tied negotiation between the three considered objectives (the contribution of all the objective functions is 33%). *S2* represents *rotation* φ_x dominated solution (p = 90%). *S3* represent *displacement* δ_z dominated solution (p = 90%). *S4* represent the *mass* dominated solution (p = 90%).

The same optimization model as previously described was further on solved by using the ARSM algorithm following the weighted sum approach. The results are represented by single values given to the model responses shown within Table III. By comparing Table III with Table II, it is concluded that better trade-offs may be obtained when solving the multi-objective problem using dedicated algorithms combined with the proposed selection method instead of reducing the problem to a single-objective definition by considering weight factors.

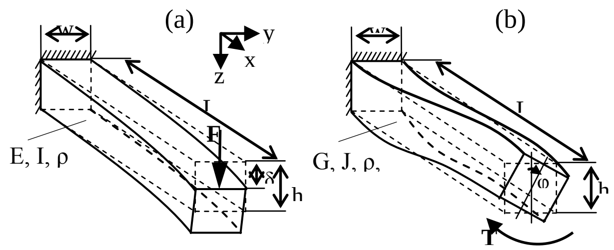


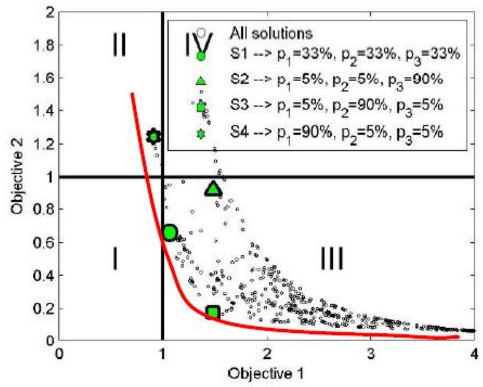
Fig. 7. Bending of a cantilever beam (a) and Torsion of cantilever beam (b) – *w* and *h* are design variables, while δ_z, φ_x and the *m* (mass of the beam) represent the responses considered as objectives to be minimized.

TABLE II: RESPONSES' VALUES AND THEIR CORRESPONDING PROPORTION FACTORS FOR DIFFERENT DESIGN SOLUTIONS OBTAINED BY USING THE PROPOSED SELECTION METHOD

Responses	Solutions							
	S1		S2		S3		S4	
	Obj. value	*p %*	*Obj. value*	*p %*	*Obj. value*	*p %*	*Obj. value*	*p %*
mass [-]	1.07	33	1.49	5	1.48	5	**0.91**	**90**
δ_z [-]	0.66	33	0.92	5	**0.18**	**90**	1.24	5
φ_x [-]	1.16	33	**0.22**	**90**	1.17	5	1.15	5
P_s [-]	5.52		1.21		1.58		10.44	
w [m]	0.045		0.087		0.038		0.049	
h [m]	0.059		0.043		0.098		0.047	

TABLE III: RESPONSES' VALUES OBTAINED BY USING THE WEIGTHED SUM APPROACH

Responses	Solutions							
	S1		S2		S3		S4	
	Obj. value	*weight*	*Obj. value*	*weight*	*Obj. value*	*weight*	*Obj. value*	*weight*
mass [-]	0.84	0.33	0.96	0.05	1.44	0.05	**0.84**	**0.9**
δ_z [-]	1.64	0.33	1.32	0.05	**0.19**	**0.9**	1.64	0.05
φ_x [-]	1.19	0.33	**0.89**	**0.9**	1.2	0.05	1.2	0.05
w [m]	0.049		0.054		0.038		0.049	
h [m]	0.042		0.044		0.095		0.042	



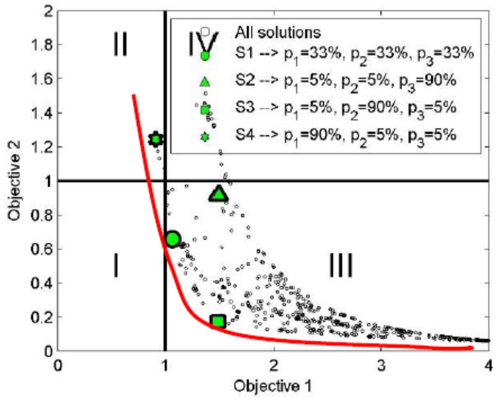


Fig. 8. Obj.1 - mass vs. Obj.2 – δ_z; Selected solutions using the proposed method.

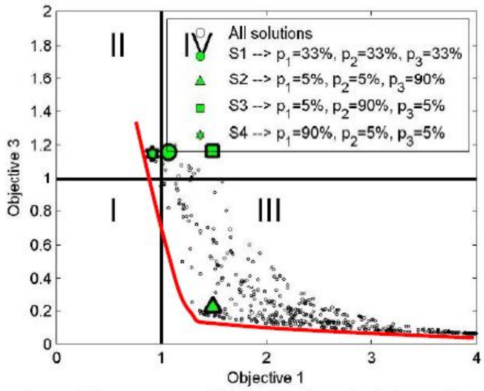


Fig. 9. Obj.1 - mass vs. Obj.3 – φ_x; Selected solutions using the proposed method.

IV. CONCLUSION

The proposed selection method allows searching within Pareto optimal sets for the most beneficial solutions by following the importance allocated for each of the objectives, using the so-called proportion factors. The procedure is therefore following a multi-objective optimisation solved by using dedicated multi-objective algorithms such as MOGA or GRSM algorithms that generates Pareto optimal solutions. Compared to the weighted sum approach, the dedicated multi-objective algorithms combined with the proposed selection method will take longer time to reach the solution. However, the main advantage of using the proposed method comes from the fact that it allows exploring the full Pareto optimal design solutions which gives the complete trade-off information for the end-decision makers and possibly the chance to select a better compromise.

ACKNOWLEDGMENT

This work was partially supported by the strategic grant POSDRU/159/1.5/S/137070 (2014) of the Ministry of Labor, Family and Social Protection, Romania, co-financed by the European Social Fund – Investing in People, within the Sectorial Operational Program Human Resources Development 2007-2013.

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